MA 3139 - Fourier Analysis and Partial Differential Equations
Objectives

Upon successful completion of this course, one should be able to:

In the area of Sequences and Infinite Series:

1. State the difference between a sequence and an infinite series, and why convergence of a sequence of terms does not guarantee convergence of the series.
2. Given an appropriate infinite series, determine whether it converges by the comparison test or integral test.
3. State the definition of the error in approximating an infinite series by finite number of terms, and describe the order of the error for a series of constants whose terms decay as $1/n^p$.
4. For a series of functions, describe the difference between pointwise and uniform convergence, and how this is related to the continuity of the limit function.

In the area of Fourier Series:

5. Determine whether a function, given either graphically or analytically, will have a Fourier series.
6. Given an appropriate function, find its Fourier series expansion.
7. Interpret the Fourier coefficients a_n and b_n in terms of amplitude, phase, and power at specific frequencies.
8. Given a Fourier series, determine from the coefficients the continuity of the function represented by the series.
9. Define the term mean square convergence, and interpret this in terms of the power represented by the terms of the Fourier series.
10. Given an appropriate even or odd function, find the Fourier half-range (sine or cosine) expansion.
11. Convert between Fourier sine/cosine, real amplitude/ phase, and complex forms, and describe the utility of each representation.
12. Use the Fourier series to find the response of a second order constant coefficient system to a periodic forcing function, and interpret the relation between the coefficients in the response and those in the input.

In the area of Partial Differential Equations:

13. Solve the wave equation in one or two-dimensional rectangular coordinates, with Dirichlet or Neumann boundary conditions, using Fourier series.
14. Given a one-dimensional wave equation in an infinite medium, express the D'Alembert solution and display graphically how terms like $f(x - ct)$ represent moving waves.
15. Interpret the eigenvalues of the wave equation in terms of fundamental modes and frequencies of vibration, and, given a one or two-dimensional wave equation, determine its fundamental frequencies and modes.

16. Explain the importance of characteristics in wave propagation, and, given a one-dimensional wave equation, find the equation of the characteristics.

17. State the form of Bessel's equation of order n, and the form of the general solution to Bessel's equation in terms of ordinary Bessel functions and Hankel functions.

18. State the form of the modified Bessel's equation of order n, and express the general solution to it in terms of modified Bessel functions.

19. Given an applicable variable coefficient second-order ordinary differential equation, convert it into a variant of Bessel's equation by the appropriate change of variables, and express the general solution in terms of Bessel functions.

20. Sketch the general behavior of the ordinary Bessel functions of order $1; 2; 3$.

21. State the orthogonality integral for Bessel functions.

22. Solve the wave equation in cylindrically symmetrical regions, and determine the fundamental frequencies and modes of propagation.

In the area of Fourier Transforms:

23. State the definition of the complex Fourier transform, and the Fourier inversion formula.

24. State conditions under which a function will have a Fourier transform, and given an appropriate function, compute its Fourier transform from the definition.

25. Using appropriate tables, determine the inverse of a given Fourier transform, to include those cases where a change of variable must be performed in order to agree with formulas in the tables.

26. State the transform formulas for the Fourier sine and Fourier cosine transforms.

27. Reduce the Fourier transform to either the sine or cosine transform in cases of appropriate symmetry.

28. Describe the relation between the Fourier transform and the Laplace transform, in terms of the solutions to second order constant coefficient ordinary differential equations.

29. Define the transfer function, and given a constant coefficient ordinary differential equation, determine its Fourier transform transfer function.

30. State the convolution theorem for Fourier transforms, and explain its importance in terms of the result from passing a signal through a sequence of linear systems.

31. Graph and interpret, in physical terms, the amplitude and phase of a complex Fourier transform.

32. Use the Fourier transform to solve the one-dimensional wave equation in an infinite region, and relate the solution obtained to the D'Alembert solution.